# Long-Term Predictive Maintenance: A Study of Optimal Cleaning of Biomass Boilers

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## Abstract

Combustion in a biomass-fired boiler causes build-up of soot, which reduces the heat transfer and decreases the efficiency of operation. In order to mitigate this natural occurrence, cleaning via soot blowing is an important maintenance action. The objective of this study is to develop long-term optimal maintenance strategies, which are model-based and specifically employ the dynamics of boiler efficiency and of anticipated heating demand, both of which are identified from empirical data. An approximate dynamic programming algorithm is set up, resulting in the optimal maintenance actions over time, so that the total operational costs of the biomass boiler plus the cleaning costs are minimised. A practical case study with real data is used to elucidate the benefits of the new approach.

*Keywords:* optimal maintenance, energy efficiency, biomass boilers, dynamic programming

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## 1. Introduction

Biomass boilers are one of the promising future avenues for heat generation [1], and have been recently deployed significantly in the United Kingdom, due to vigorous governmental subsidies [2]. However, they have arisen several concerns, ranging from hardware design [3] to dynamic control [4]. The main reason is that biomass boilers have longer response times than gas or coal boilers. Poor hardware design and installation, as well as inappropriate and overly reactive control, may result in frequent on/off switching that is very inefficient.

Another difficulty is what is known as fouling. On the surfaces of the heat exchangers, soot is accumulated during operation. It causes worse heat transfer to water and more heat is lost in the exhaust air. Fouling in biomass boilers may be more problematic than in more traditional types of boilers because the biomass particles are typically more volatile than e.g. coal ones.

This article addresses the problem of mitigating fouling in biomass boilers by deriving optimal maintenance strategies. Specifically, it determines when is the most appropriate moment to clean the heat exchanger against the cost of biomass, the cost of cleaning itself, the dynamics of fouling, and a predicted heating demand. Considering all these aspects explicitly and within the same framework of dynamic programming (as already outlined in [5]) makes the present approach unique and novel.

[6, 7] have examined the option of inference over the system based on expert knowledge, where manually specified rules are applied to current data. Others have dealt with the optimization of the boiler maintenance already: [8] assume the presence of soot blowers and use a set of neural networks to capture the behaviour of the system. The result of these neural networks is evaluated by a set of fuzzy logic rules. More recently, [9] apply more accurate first-principle modelling and the ultimate decision is made about the duration and timing of operation of soot blowers. Similarly [10] focus on soot blowers: they combine neural network modelling with optimisation based on sequential quadratic programming. None of these articles considers long-term prediction of demand as an important factor for the decision making about maintenance actions. The need for that becomes more evident in the case of boilers without soot blowers, where the maintenance (cleaning) requires manual work that is more expensive in contrast to operation of the soot blowers.

This article has the following structure: Section 2 defines the problem formally. Then, model construction is discussed in Section 3. The model is used for optimisation via dynamic programming in Section 4. The algorithms are demonstrated on a case study with real data in Section 5. Finally, Section 6 concludes the article.

## 2. Problem Formulation

### *2.1. Description of the Physical System*

We consider a system that involves a biomass boiler with a hopper within a building, as shown in Figure 1. We consider three mass flows: the biomass delivery $\dot{m}_{fuel}$ [kg/h], the air flowing through the combustion chamber $\dot{m}_{air}$ [kg/h], and the heated water flowing out of the boiler $\dot{m}_{water}$ [kg/h]. In this

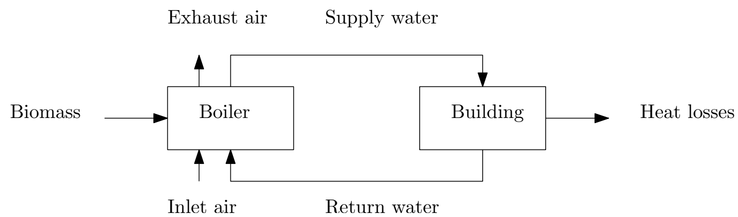


Figure 1: Layout of the physical system.

article, we will assume $\dot{m}_{water}$ and $\dot{m}_{air}$ to be constant. This assumption is valid especially in smaller installations where these two quantities are not subject to control. On the other hand, we consider $\dot{m}_{fuel}$ to be a measurable quantity that is controlled to satisfy the heat demand in the building.

There are several energy flows related to the mass flows. First, the heat produced by burning the biomass can be expressed as

$$P_{in} = H_{fuel} \cdot \dot{m}_{fuel}, \tag{1}$$

where $H_{fuel}$ [J/kg], is the amount of heat released by total combustion of unit of fuel, which is assumed to be known. Thus, $P_{in}$ [kW] is observable. A part of the generated heat is delivered to the building, while another is lost with the exhaust air. We will denote them as $P_{out}$ [kW] and $P_{loss}$ [kW] respectively and it holds that

$$P_{in} = P_{out} + P_{loss}. \tag{2}$$

The output power can be calculated as

$$P_{out} = c_{water} \cdot (T_{sw} - T_{rw}) \cdot \dot{m}_{water}, \tag{3}$$

where $c_{water}$ [J/kgK] is the specific heat capacity of water, $T_{sw}$ [K] is the temperature of supply water and $T_{rw}$ [K] is the temperature of return water. All the quantities on the right-hand side are known or observable. Hence, $P_{out}$ can be directly inferred from the measurements.

The proportion of the heat delivered to the building is called combustion efficiency (a dimensionless quantity) and is defined as

$$\eta = \frac{P_{out}}{P_{in}}. \tag{4}$$

The efficiency decreases during the use of the boiler. In other words, more fuel is required to cover a particular heating demand. However, we assume that the boiler can be subject to cleaning (either manual or automatic by soot blowers), which brings it back to the original state. These cleaning costs $p_{cln}$ are in monetary units, in this work [EUR]. Another price is $p_{fuel}$ [EUR/kg], the price of a unit of biomass, which can be transformed easily to price of a unit of input energy as $p_{en} = p_{fuel}/H_{fuel}$ [EUR/J] .

### *2.2. General Requirements*

Before we define the problem formally, we can intuitively claim some properties that a system for optimal boiler maintenance ought to satisfy:

- The total operational costs, for a mass of fuel $m_{fuel}$ [kg], are $p_{op} = p_{cln} + m_{fuel}p_{fuel}$.
- If $m_{fuel}p_{fuel} \gg p_{cln}$, it is beneficial to perform maintenance as often as possible, such that $m_{fuel}$ is kept at a minimum.
- If $m_{fuel}p_{fuel} \ll p_{cln}$, it is beneficial to do no maintenance.
- If there are almost no heat losses to be compensated in the building, i.e. $P_{out} \approx 0$, no maintenance is required, regardless to the state of boiler.
- If there is extreme (hypothetically infinite) demand on compensation of the heat losses, i.e. $P_{out} \approx \infty$, the maintenance is to be very frequent (hypothetically continuous).

The decisions about maintenance actions depend on the heat demand $P_{out}$. Specifically, the decisions will be influenced not only by the current heat demand, but also by the forecasted heat demand. This fact motivates the formalisation of the problem as a dynamical system - this is in contrast to the cited state-of-art approaches.

### *2.3. Problem Formalisation as a Dynamical System*

We formalise the process of boiler fouling with a dynamical system. The time step can be arbitrary, but we select rather longer periods such weeks. There are three reasons for that: (i) boiler maintenance actions are considered in long time horizons; (ii) considering short periods such as in the order of hours (the horizon used for thermal dynamics) makes the problem computationally more complex; (iii) the use of longer time intervals can help with aggregation: we can indeed consider the weekly heating demand as something that can be predicted for months ahead, but we cannot do the same with hourly consumptions (the precision would be extremely low).

We will use $n = 1, \ldots, N$ as the indices of time intervals. For example, $n$ is index of a week and the horizon has $N$ weeks. The length of a time interval $l$ (e.g. number of seconds in a week) leads to consider the aggregation in time of the considered quantities. The aggregated input power forms the aggregated input heat $Q_{in}[n]$ [J] as

$$Q_{in}[n] = \int_{(n-1)l}^{nl} P_{in}(t)dt; \tag{5}$$

the aggregated output heat $Q_{out}[n]$ [J] is formed similarly as

$$Q_{out}[n] = \int_{(n-1)l}^{nl} P_{out}(t)dt, \tag{6}$$

and the aggregated heat loss is $Q_{loss}[n]$ [J]. Using these quantities, we can write the aggregated efficiency $\eta[n]$ as

$$\eta[n] = \frac{Q_{out}[n]}{Q_{in}[n]}. \tag{7}$$

With respect to the identities (2) and (7), it is sufficient to model the state as vector $x[n] = (\eta[n], Q_{out}[n])^T$, leaving out $Q_{in}[n] = Q_{out}[n]/\eta[n]$ and $Q_{loss}[n] = Q_{in}[n] - Q_{out}[n]$.

The dynamical system is subject to actions $u$ (a dimensionless quantity). At the end of each period $n$, we can decide to either carry out the maintenance action, i.e. $u[n] = 1$, or not, i.e. $u[n] = 0$.

The so-called single transition costs $C(x[n], u[n])$ [EUR] express the cost incurred during one period, comprising the cost of operation and of maintenance as

$$C(x[n], u[n]) = p_{en} \cdot \frac{Q_{out}[n]}{\eta[n]} + p_{cln} \cdot u[n]. \tag{8}$$

The dynamics of the system is given by the state-evolution model $x[n+1] = f\left(x[n], u[n]\right)$. The construction of the mapping $f$ is not straightforward and we deal with it in Section 3.

### 2.4. Problem Statement

Based on the available data, we want to schedule maintenance actions optimally. Considering $X$ to be the set of all possible states and $U$ the set of all possible actions, we want to calculate decision rules $\pi[n] : X \to U$ for all $n = 1, \dots, N-1$ so that the total cost

$$\sum_{n=1}^{N} C(x[n], u[n]) \tag{9}$$

is minimal, subject to the conditions on the transitions

$$x[n+1] = f(x[n], u[n]) \tag{10}$$

and the application of the action

$$u[n] = \pi[n](x[n]). \tag{11}$$

We solve the problem in two steps. First, we identify the state-evolution model in Section 3. Then, we calculate the optimal strategy by means of dynamic programming in Section 4.

Figure 2 summarises the structure of model and control signals. The plant/process has states consisting of efficiency and heat demand that are observed by the controller. The controller applies the current decision rule on the current observation and decides about next action (input), i.e. whether the boiler is to be cleaned or not. This influences the state of the plant. The state of the plant is also influenced by the reference heating demand. Finally, the cost function quantifies the cost of one-step operations.

## 3. Modeling the System Dynamics

In this section, we discuss the model dynamics $x[n+1] = f(x[n], u[n])$. The system state has two components: the first one is the efficiency level, the second one is the heating demand. Both of them capture the most essential dynamics for the optimal maintenance problem. The construction of the corresponding models is described in the following.

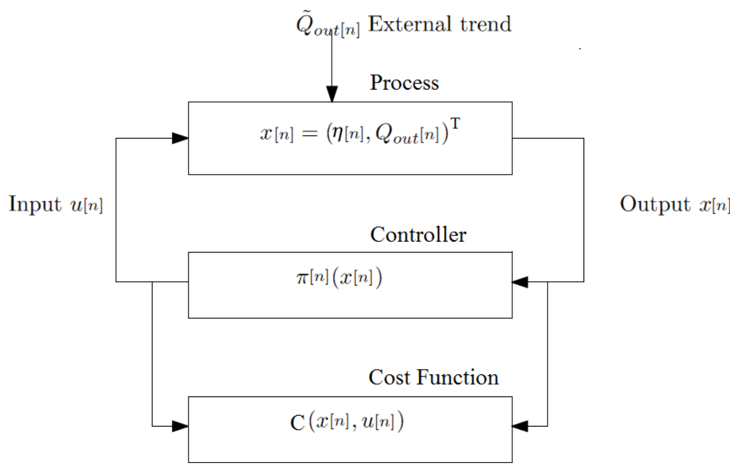


Figure 2: Structure of model and control signals.

### *3.1. Heating Demand*

We discuss how to model the demanded heat supply $Q_{out}[n]$. As mentioned above, $Q_{out}[n]$ is the amount of heat that is required to compensate the heat losses of the building to maintain the indoor comfort at a given level. For the sake of simplicity, we assume that the heating demand is always met.

The heating demand does not depend on the state of the boiler, because it is rather a property of the building and would be the same even if we would equip the building by a completely different heating system. Thus, demand does not depend on the boiler efficiency $\eta[n]$ nor on the maintenance action $u[n]$. However, $Q_{out}[n+1]$ possibly depends on $Q_{out}[n]$, because we can assume that the heating demand will not change rapidly, and on $n$ because the heating demand has a long-term trend, which depends especially on the seasonal influences. We adopt the model

$$Q_{out}[n+1] = \gamma Q_{out}[n] + (1-\gamma)\tilde{Q}_{out}[n+1], \tag{12}$$

where $\tilde{Q}_{out}[n+1]$ is the long-term trend of $Q_{out}$ and $\gamma \in [0, 1]$ is a parameter. A similar approach combining the autoregressive behaviour and long-term trends was employed also in [11].

Given the model structure (12), two steps are to be carried out: (i) to construct the long-term trend $\tilde{Q}_{out}[n+1]$ and (ii) to estimate the parameter $\gamma$.

The construction of the long-term trends of heat demand (and energy demand in general) has been addressed by many authors, e.g. [12, 13, 14].

Any of the mentioned methods can be adopted and possibly tailored. Since some of the periods will have no demand (especially in warm season of the year), we construct the auxiliary long-term trend $r[n]$ [J] as follows:

1. First, we create a classifier $h$ that will indicate whether any heat demand is considered for a given $n$: this function takes value one during the heating season, zero otherwise.
2. Then, the trend $r$ is calculated from data when the heating demand is positive. For that purpose Gaussian process, local regression, or (as in our case) frequency-domain linear regression [15] can be adopted.

The prediction of this trend has been examined extensively [16, 17, 18]. Having identified $h$ and $r$, the trend is defined as

$$\tilde{Q}_{out}[n] = \begin{cases} r[n] & \text{if } h[n] = 1 \\ 0 & \text{otherwise.} \end{cases} \tag{13}$$

Further, in order to estimate $\gamma$, we can use a de-trending term

$$\Delta[n] = Q_{out}[n] - \tilde{Q}_{out}[n] \tag{14}$$

and employ standard tools for autoregressive modelling. Considering the constraint $\gamma \in [0, 1]$, we can solve an optimisation problem with the objective function

$$\gamma^* = \arg \min_{\gamma \in [0,1]} \sum_{n=2}^{D} \Big( Q_{out,d}[n] - \gamma Q_{out,d}[n-1] - (1-\gamma)\tilde{Q}_{out,d}[n] \Big)^2 \tag{15}$$

considering a data set $\left\{ Q_{out,d}[n], \tilde{Q}_{out,d}[n] \right\}_{n=1}^{D}$ where $D$ is number of samples (e.g. observed weeks)[1].

## *3.2. Fouling Process and Efficiency*

In this section, we discuss the dynamics of the efficiency term $\eta[n+1]$, which depends on:

- The most recent efficiency $\eta[n]$. This efficiency corresponds to the level of accumulated soot.

[1]Throughout this paper we denote data variables with $x_d$ and model variables as $x$.

- The most recent operation. If there is no operation, i.e. the boiler is not used, and no biomass is burned in it, then the efficiency level remains the same. If the operation is intensive, i.e. the boiler burns a large amount of biomass and generates much soot, the efficiency decreases rapidly. The operation can be quantified in terms of delivered heat $Q_{out}[n]$, generated input heat $Q_{in}[n]$, or the number of switches.

- The most recent action $u[n]$. If the boiler is cleaned and all soot is removed, its efficiency improves, possibly to the original level.

We have adopted an approximate model that satisfies these conditions:

$$\eta[n+1] = \begin{cases} \eta[n] + \alpha Q_{out}[n] & \text{if } u[n] = 0 \\ \eta\text{max} & \text{otherwise.} \end{cases} \tag{16}$$

Since we assume $Q_{in}$ and $Q_{out}$ to be measurable, we can observe $\eta$ as well. The unknown boiler fouling parameter $\alpha$ is assumed to be negative as it captures the negative impact of the operation $Q_{out[n]}$ on the efficiency $\eta[n+1]$. It can be easily fitted by standard tools for linear regression, considering $\eta[n] - \eta[n-1]$ as the output variable and $Q_{out}[n-1]$ as the input variable.

There are several other models for efficiency that are based on first principles [3, 8], however they cope especially with short-term dynamics, which is not the focus of this article.

## 4. Optimal Maintenance via Dynamic Programming

Having a specific model for the state evolution $x[n+1] = f(x[n], u[n])$, we can address the problem of the optimal maintenance by means of dynamic programming [19]. This employs a so-called value function $V$ [EUR], which is updated backwards, starting from the very last time interval

$$V[N](x[N]) = \inf_{u[N]\in U} C(x[N], u[N]). \tag{17}$$

For the remaining time indices $n = N-1, N-2, \ldots 1$, the value is calculated recursively as

$$V[n](x[n]) = \inf_{u[n]\in U} C(x[n], u[n]) + V[n+1](f(x[n], u[n])). \tag{18}$$

Having the value functions $V_{[n]}$, the decision rule is defined as

$$\pi[n](x[n]) = \arg \inf_{u[n]\in U} C(x[n], u[n]) + V[n+1](f(x[n], u[n])). \tag{19}$$

This optimisation is straightforward because set $U$ has two elements only. More intuitively, (19) can be qualitatively interpreted as follows:

$$\text{Next action for given } x[n] = \begin{cases} \text{cleaning} & \text{if it is cheaper in the long term} \\ \text{do nothing} & \text{otherwise.} \end{cases} \tag{20}$$

To implement a program calculating this optimal strategy, we discretise the state space $X \subset \mathbb{R}^2$ so the value functions $\hat{V}[n]$ are computed over a discrete domain [20]. We cover the state space by disjoint set of rectangles $\bigcup_{i=1}^{I} X_i = X$. Specifically, we consider the range for the efficiency $[\eta^{\min}, \eta^{\max}]$, sliced into $I_1$ intervals, and for the output heat $[Q_{out}^{\min}, Q_{out}^{\max}]$, sliced into $I_2$ intervals. The number of the points of the discrete domain is thus $I = I_1 \cdot I_2$.

Having calculated the value function $\hat{V}[n](x_i)$ at the centre of these rectangles $x_i \in X_i$, we approximate the value function for any $x \in X$ by the value of the corresponding centre, i.e.

$$\hat{V}[n](x) = \hat{V}[n](x_i), \tag{21}$$

where $x \in X_i$.

The calculation is summarized in Algorithm 1. The algorithm has the following inputs: the state-evolution models $f$, discretization of the input space $(X_i, x_i)_{i=1}^{I}$, the set of actions $U$, and the cost function $C$. On line 2, the value function approximation is initiated to zeros for an hypothetical time interval $N + 1$. This initialization assures that the very first step of the recursion will be carried out according to (17). Lines 3 to 15 describe the backward recursion. In each step of the recursion, the value function is calculated for all centers of the rectangles, see lines 4 to 14. Lines 5 to 13 implement the minimization (18), using the approximation (21) implemented on lines 7 to 9.

### *4.1. Special Case: No Autoregression*

In Section 3, we discussed the modelling of the heating demand $Q_{out}$. If the estimation procedure results in no autoregression term, specifically $\gamma = 0$, then we can modify the model as follows: $Q_{out}[n] = \tilde{Q}_{out}[n]$ is constant. We then introduce a modification in the change of the gridding. Namely, the gridding of $[\eta^{\min}, \eta^{\max}]$ remains the same, and is sliced into $I_1$ intervals. On the other hand, the gridding for $Q_{out}$ is implemented so that the second coordinates of centres of the rectangles are the forecasted values $\tilde{Q}_{out}[n]$.

**Algorithm 1** Calculation of value functions

1: **procedure** CALCULATEV($f, (X_i, x_i)_{i=1}^{I}, U, C$)
2: $\quad \hat{V}[N](x_i) \leftarrow 0 \quad \forall i = 1, \ldots, I$
3: $\quad$ **for** $n = N-1, \ldots, 1$ **do**
4: $\quad\quad$ **for** $i = 1, 2, \ldots, I$ **do**
5: $\quad\quad\quad \hat{V}[n](x_i) \leftarrow \infty$
6: $\quad\quad\quad$ **for** $u \in U$ **do**
7: $\quad\quad\quad\quad x' \leftarrow f(x_i, u)$
8: $\quad\quad\quad\quad i' \leftarrow$ index of the rectangle where $x' \in X_{i'}$
9: $\quad\quad\quad\quad V_{tmp} \leftarrow C(x_i, u) + \hat{V}[n+1](x_{i'})$
10: $\quad\quad\quad\quad$ **if** $V_{tmp} < \hat{V}[n](x_i)$ **then**
11: $\quad\quad\quad\quad\quad \hat{V}[n](x_i) \leftarrow V_{tmp}$
12: $\quad\quad\quad\quad$ **end if**
13: $\quad\quad\quad$ **end for**
14: $\quad\quad$ **end for**
15: $\quad$ **end for**
16: **end procedure**

It then holds that

$$V[n](x[n]) = V[n](\eta[n], Q_{out}[n]) = V[n](\eta[n], \tilde{Q}_{out}[n]). \tag{22}$$

This implies that the second argument is fixed. Thus, there is no need to approximate $V_{[n]}$ for other $Q_{out}[n] \neq \tilde{Q}_{out}[n]$. Therefore, we can modify line 4 so that we iterate not for all $i = 1, 2, \ldots, I$, but only for those $i = 1, 2, \ldots, I$ where the second component of the centre $x_i \in X_i$ is equal to heat demand for $n$, i.e. $x_{i,2} = \tilde{Q}_{out}[n]$.

Treating this case separately will increase the precision because there is no approximation in terms of $Q_{out}$. Moreover, it results in faster calculation because we process only $I_1$ rectangles in the approximation. The tabular representation of the optimal actions can also be practically reduced, as the value functions are not calculated for most of the combinations.

## 5. Case Study

For the case study we have used data from a building in Spain, equipped by a biomass boiler. Based on almost one year of data, we plan to create a maintenance strategy for the following 10 years. Considering the length of a time interval to be 1 week, the chosen horizon results in $N = 521$ weeks.

### 5.1. Specification of Parameters

The general parameters are: specific heat capacity of water $c_{water} = 4180$ [J/kgK], heating value of biomass $H_{fuel} = 2.2 \times 10^7$ [J/kg], price of biomass[2] $p_{fuel} = 0.27$ [EUR/kg], price of maintenance[3] $p_{cln} = 108.9$ [EUR], mass flow of the water $\dot{m}_{water} = 2.15 \times 10^4$ [kg/h].

### 5.2. Data Preprocessing

The measured data are from October 2013 to September 2014. The available measurements are: supply water temperature $T_{sw}$, return water temperature $T_{rw}$, water mass flow $\dot{m}_{water}$, biomass consumption $\dot{m}_{fuel}$. The data are measured with a sampling time of 15 minutes, i.e. we have $q = 4 \cdot 24 \cdot 7 = 672$ records per week. Let us index the sampled data by $l$ and use temperatures $T_{rw,d}[l]$ [K], $T_{sw,d}[l]$ [K], which represent the mean value of $T_{rw}$, $T_{sw}$ within the sample, and absolute masses $m_{fuel,d}[l]$ [kg], $m_{water,d}[l]$ [kg] each computed as integral of the corresponding mass flow over sampling interval $l$.

To obtain the values for $x[n]$ for the construction of state-evolution models we have used the following formulas:

$$Q_{out,d}[n] = \sum_{l=(n-1)q+1}^{nq} c_{water} \cdot (T_{sw,d}[l] - T_{rw,d}[l]) \cdot m_{water,d}[l]\,, \tag{23}$$

$$Q_{in,d}[n] = \sum_{l=(n-1)q+1}^{nq} H_{fuel} \cdot m_{fuel,d}[l]\,, \tag{24}$$

$$\eta_d[n] = \frac{Q_{out,d}[n]}{Q_{in,d}[n]}. \tag{25}$$

### 5.3. Results of Modelling

As described in Section 3, we model the dynamics of the system in two steps: first the model of the heating demand, then the model of fouling.

[2] http://www.avebiom.org/es/noticias/News/show/precios-del-pellet-en-espana-653
[3] http://www.tucalderabarata.es/reparacion-de-calderas/

The first step of heating demand modelling is the construction of the classifier $h$. Based on an inspection of available data, we consider the period from the first week in November to the last week of February as a heating season, i.e. $h[n] = 1$, with the exception of the last week in December and first week in January, when the building is not used nor heated. Otherwise, no heating demand is considered, i.e. $h[n] = 0$. For this classification as well as for the next calculation, we will use $t[n]$ as the expression of the absolute time in days, as Matlab implements the date-time values.

For the weeks when the building was heated, the trend $r[n]$ is fitted using frequency-domain linear regression [15] to the model

$$\begin{aligned} r[n] =& \beta 0 + \beta 1 \cos\left(\frac{2\pi t[n]}{365}\right) + \beta 2 \sin\left(\frac{2\pi t[n]}{365}\right) \\ &+ \beta 3 \cos\left(\frac{4\pi t[n]}{365}\right) + \beta 4 \sin\left(\frac{4\pi t[n]}{365}\right). \end{aligned} \tag{26}$$

The model was fitted by `lscov` Matlab function using 14 data samples. The parameters are shown in Table 1.

Table 1: Results of the modelling - parameters

| Parameter | $\beta 0$ | $\beta 1$ | $\beta 2$ | $\beta 3$ | $\beta 4$ |
|---|---|---|---|---|---|
| Value ($\times 10^5$) | -2.8122 | -2.1200 | 3.6246 | 0.6391 | 1.0429 |

The standard deviation from the trend was 3738.8 kWh/week, i.e. 21.69% of the mean. We tried to explain this by the use of an autoregressive term, however the result was $\gamma = 0$ which leads to the special case discussed in Subsection 4.1. The final model as well as the underlying data is shown in Figure 3.

The boiler fouling parameter $\alpha$ was estimated also by standard `lscov` Matlab function. The result was

$$\hat{\alpha} = -7.4160 \times 10^{-7} \, [\text{kWh}^{-1}]$$

which corresponds to 17.91% decrease of the efficiency for a heating season without maintenance. This can be seen from Figure 4 which depicts the measured data versus the resulting model. The fitted original value $\hat{\eta}^{\max} = 0.6396$.

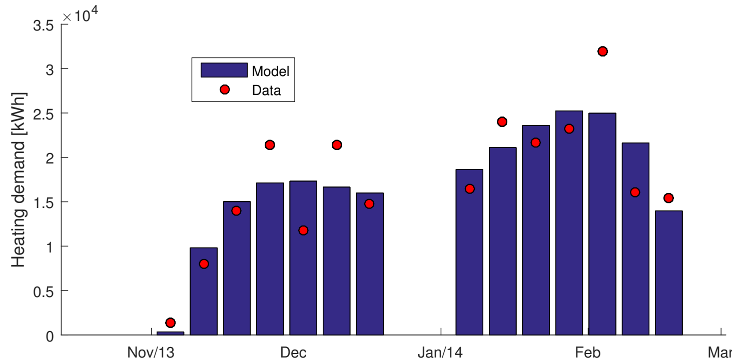


Figure 3: Heating demand (data vs. model).

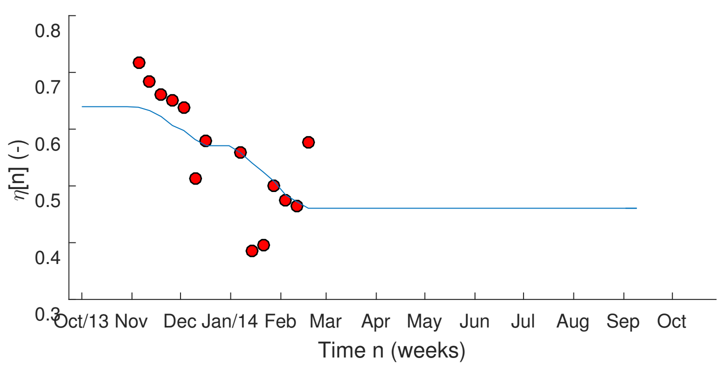


Figure 4: Fouling process and corresponding efficiency model (data vs. model).

### 5.4. Results of Optimization

Since the identification resulted in $\gamma = 0$, we adopted the approach discussed in Subsection 4.1. We used the discretization to $I_1 = 100$ intervals, considering the efficiencies to range from $\eta^{\min} = 0.20$ to the detected $\eta^{\max} = 0.6396$.

The resulting strategy has the shape as shown in Figure 5. For each time $n$ and efficiency level $\eta[n]$, the optimal action is given. Where the colour is white, there is no maintenance, i.e. $\pi[n](x[n]) = 0$. Where the colour is blue (dark), the maintenance action is carried out, i.e. $\pi[n](x[n]) = 0$. Note that the resulting strategy satisfies the general requirements, as outlined in Section 2.2: the maintenance depends on the heat demand. We can also observe decreased willingness to clean the boiler in the last year of the prediction horizon. This is also an expected behaviour: if there is no heating assumed anymore then the maintenance does not make sense. Detailed version is provided in Figure 6. We can observe one interesting detail: the last month of the prediction horizon is December 2025. The maintenance actions are carried out in case of very low efficiency only. It is contrasting to December 2024 where the efficiency can be still high and the maintenance action is proposed. The reason for this difference is the following: the strategy decides myopically in the end because in the short-term perspective the maintenance is relatively more expensive than achieved savings on fuel.

The visual inspection indicates that the strategy can be interpreted as follows: if the efficiency is below a threshold, specifically, and if there is a heating demand in the upcoming period, i.e. $Q_{out}[n+1] > 0$, then the maintenance is to be carried out. We can then define an optimal condition-based maintenance strategy by setting the threshold to a fixed value $\eta^{thr} = 0.58$ based on the visual inspection of Figure 5. Note that this maintenance strategy differs from standard condition based maintenance significantly because it uses a threshold that is not given by an expert estimate, but as a result of explicit optimisation that captures all available information. As such, our procedure could provide an automatic update of such a threshold, which would otherwise be difficult to set based on expert opinion.

Table 2 summarizes the results of the dynamic programming compared to annual or semi-annual maintenance. We can see that the dynamic programming is much better than regular approaches.

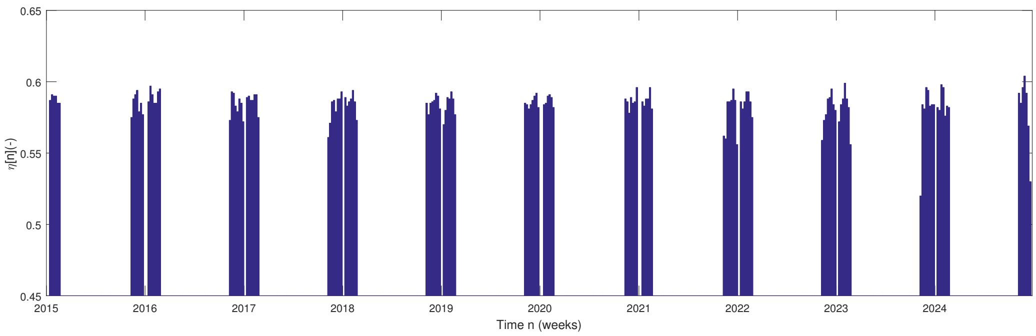


Figure 5: Calculated decision strategy: values of $\eta[n]$ when a maintenance action is to be carried out.

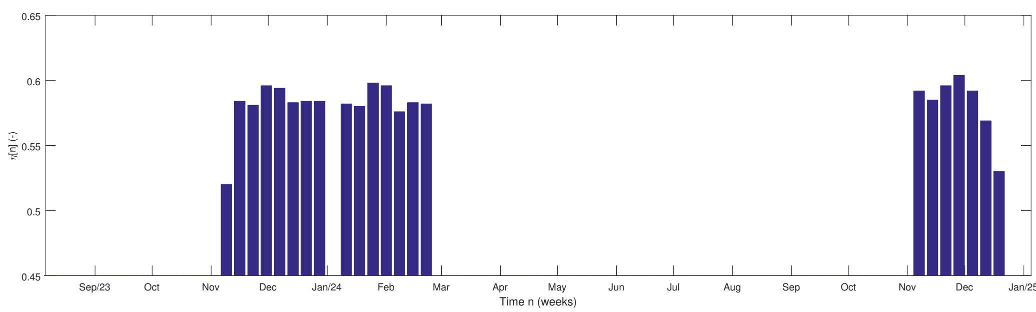


Figure 6: Detail on last two years.

Table 2: Results

| Strategy | Total cost [EUR] | Total actions | Savings by Dynamic Programming |
|---|---:|---:|---:|
| Dynamic programming | 5.0675e+04 | 26 | 0% |
| Annual cleaning | 5.5385e+04 | 10 | 8.50% |
| 6month cleaning | 5.6474e+04 | 20 | 10.27% |
| No maintenance | 1.2949e+05 | 0 | 60.87% |

## 6. Conclusions

The present article has demonstrated the capability of dynamic programming as a tool for the optimal predictive maintenance on practical models obtained from real data. The described methodology is motivated not only on the state of the equipment, but also on the long-term trends of the heating demand, which is novel considering the state-of-art. The approach has been applied to a biomass boiler at a Spanish school and has highlighted possible energy savings when compared to standard maintenance strategies.

There are many open areas for further development. The article has adopted relatively basic tools for the modelling (heating demand, fouling process) as well as for the optimisation (discretised dynamic programming). Advanced modelling and optimisation tools can enrich the approach. These methods can be considered not only in the batch implementation, as described here, but also in an online set-up. It may be also beneficial to explore the possibility to consider retrofit as a possible action to optimise not only the maintenance, but also the procurement of new equipment. A related challenge is to explore the reasonable length of the horizon as the optimal strategy exhibit relatively periodic behaviour. Finally, the cost model may incorporate also discomfort monetisation [21], leading to a number of interesting technical questions.

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## 7. References

[1] R. Saidur, E. Abdelaziz, A. Demirbas, M. Hossain, S. Mekhilef, A review on biomass as a fuel for boilers, Renewable and Sustainable Energy Reviews 15 (5) (2011) 2262 – 2289.

[2] M. Perry, F. Rosillo-Calle, Recent trends and future opportunities in UK bioenergy: Maximising biomass penetration in a centralised energy system, Biomass and Bioenergy 32 (8) (2008) 688 – 701.

[3] A. Donoghue, The dynamic modelling of biomass boiler systems: A training tool, Master's thesis, University of Strathclyde (2012).

[4] M. Gölles, S. Reiter, T. Brunner, N. Dourdoumas, I. Obernberger, Model based control of a small-scale biomass boiler, Control Engineering Practice 22 (2014) 94 – 102.

[5] J. Berka, K. Macek, Effective maintenance of stochastic systems via dynamic programming, in: Proceedings of 19th Technical Computing Prague Conference, 2011.

[6] N. Sarunac, C. Romero, Sootblowing operation: the last optimization frontier, in: 29th International Technical Conference on Coal Utilization & Fuel Systems, 2004.

[7] B. Widarsson, C. Karlsson, E. Dahlquist, Bayesian network for decision support on soot blowing superheaters in a biomass fuelled boiler, in: Probabilistic Methods Applied to Power Systems, 2004 International Conference on, 2004, pp. 212–217.

[8] L. Romeo, R. Gareta, Fouling control in biomass boilers, Biomass and Bioenergy 33 (5) (2009) 854 – 861.

[9] S. Yuanhao, W. Jingcheng, L. Zhengfeng, On-line monitoring of ash fouling and soot-blowing optimization for convective heat exchanger in coal-fired power plant boiler, Applied Thermal Engineering 78 (2015) 39 – 50.

[10] L. Pattanayak, S. Ayyagari, J. Sahu, Optimization of sootblowing frequency to improve boiler performance and reduce combustion pollution, Clean Technologies and Environmental Policy 17 (7) (2015) 1897–1906.

[11] Z. Dilaver, L. Hunt, Modelling and forecasting turkish residential electricity demand, Energy Policy 39 (6) (2011) 3117–3127.

[12] D. Leith, M. Heidl, J. Ringwood, Gaussian process prior models for electrical load forecasting, Probabilistic Methods Applied to Power Systems (2004) 112–117.

[13] K. Mařík, Z. Schindler, P. Stluka, Decision support tools for advanced energy management, Energy 33 (6) (2008) 858–873.

[14] S. Mirasgedis, Y. Sarafidis, E. Georgopoulou, D. Lalas, M. Moschovits, F. Karagiannis, D. Papakonstantinou, Models for mid-term electricity demand forecasting incorporating weather influences, Energy 31 (2) (2006) 208–227.

[15] R. Shumway, D. Stoffer, Time series analysis and its applications: with R examples, Springer Science & Business Media, 2010.

[16] E. González-Romera, D. Carmona, M. Jaramillo-Morán, Monthly electric energy demand forecasting with neural networks and fourier series, Energy Conversion and Management 49 (11) (2008) 3135–3142.

[17] L. Suganthi, A. Samuel, Energy models for demand forecastinga review, Renewable and Sustainable Energy Reviews 16 (2) (2012) 1223–1240.

[18] H. Zhao, F. Magoules, A review on the prediction of building energy consumption, Renewable and Sustainable Energy Reviews 16 (6) (2012) 3586 – 3592.

[19] D. Bertsekas, Dynamic programming and optimal control, no. 2 in 1, Athena Scientific Belmont, MA, 1995.

[20] L. Buşoniu, R. Babuška, B. Schutter, D. Ernst, Reinforcement Learning and Dynamic Programming Using Function Approximators, 1st Edition, CRC Press, Inc., Boca Raton, FL, USA, 2010.

[21] X. Fang, M. Bianchi, C. Christensen, Monetization of thermal comfort in residential buildings, in: Proceedings of 2012 ACEEE Summer Study on Energy Efficiency in Buildings, American Council for an Energy-Efficient Economy, 2012, pp. 64–75.